

$$f(n) = \underline{3n^3} + bn^2 + n + 5 \Rightarrow O(n^3)$$

$$O(n^2) \quad \begin{array}{l} n=1000 \rightarrow 5 \text{ seconds} \\ 3\times \left(\begin{array}{l} n=3000 \rightarrow 45 \text{ seconds} \\ ? \end{array} \right. \end{array}$$

↓ 9x

[1, 2, 3, 4, 5]

1 + 2 + 3 + 4 + 5

$n(3)$

1 + n + 3 n + 4

$4n + 5 \Rightarrow O(n)$

```
def stddev(data):
```

```
    result = 0.0
```

```
    average = mean(data)
```

```
    for elem in data:
```

```
        result += (elem - average) ** 2
```

```
    return math.sqrt(result / (len(data) - 1))
```

list $n \Rightarrow$ length of the list
1
list not ops

$\sim \text{sum(data)} / \text{len(data)}$

1 1

1

4

$n(n+3)$

1 + $n(n+3)$ + 4

$n^2 + 3n + 5 \Rightarrow O(n^2)$

```
def stddev2(data):
```

```
    result = 0.0
```

```
    for elem in data:
```

```
        result += (elem - mean(data)) ** 2
```

```
    return math.sqrt(result / (len(data) - 1))
```

4

n

What is the big-O asymptotic bound of

$$f(n) = \cancel{3n^2} + \cancel{4n} + \cancel{2}$$

- A. $O(n)$
- B. $O(n^2)$
- C. $O(n^2 + n)$
- D. $O(3n^2 + 4n + 2)$

Answer: B

With big-O we only retain the term with the highest growth rate and drop all constants.

Which of the following functions has the same big-O asymptotic bound as

$$f(n) = \cancel{2n^3} + \cancel{5n} + 2$$

$O(n^3)$

A. $2n^2 + 5n + 2$

B. $n^3 + \cancel{2n^2}$ $O(n^3)$

C. $n(2n^3 + 5n + 2)$

D. 2

Answer: B

With big-O we only retain the term with the highest growth rate and drop all constants.

```
numbers = list(range(100))  
the_max = max(numbers)
```

What is the time complexity of the Python max function (example above):

- A. $O(1)$
- ~~B. $O(100)$~~
- ☒ C. $O(n)$
- D. $O(n \log n)$
- E. $O(n^2)$

Answer: C

To compute the max, we need to examine all n items in the list. The time complexity is independent of any particular input, i.e., we describe it in terms of the size of the input as it grows arbitrarily large.

```

for i in range(n):
    for j in range(n):
        val = 0.0
        for k in range(n):
            val += x[i][k] * y[k][j]
        res[i][j] = val

```

$$f(n) \sim n^i (n^j (1 + n^k (2) + 1))$$

$$O(n^3)$$

The above code implements matrix multiply on two $n \times n$ matrices (stored as lists of lists). What is the time complexity of this algorithm?

- A. $O(1)$
- B. $O(n)$
- C. $O(n \log n)$
- D. $O(n^2)$
- E. $O(n^3)$

Answer: E

Since the 3 loops are nested, we will do $n * n * n$ multiplications for $O(n^3)$

$$\begin{array}{r}
 437 \\
 - 256 \\
 \hline
 181 \\
 - 128 \\
 \hline
 53 \\
 - 32 \\
 \hline
 21 \\
 - 16 \\
 \hline
 5 \\
 - 4 \\
 \hline
 1
 \end{array}$$

$$437 = 256 + 128 + 32 + 16 + 4 + 1$$

$$2^8 \quad 2^7 \quad 2^5 \quad 2^4 \quad 2^2 \quad 2^0$$

$$110110101$$

$$\begin{array}{r}
 111 \\
 10111 \\
 + 00101 \\
 \hline
 11100
 \end{array}$$

$$16 + 4 + 2 + 1 \Rightarrow 23$$

$$4 + 1 \Rightarrow 5$$

$$16 + 8 + 4 \Rightarrow 28$$

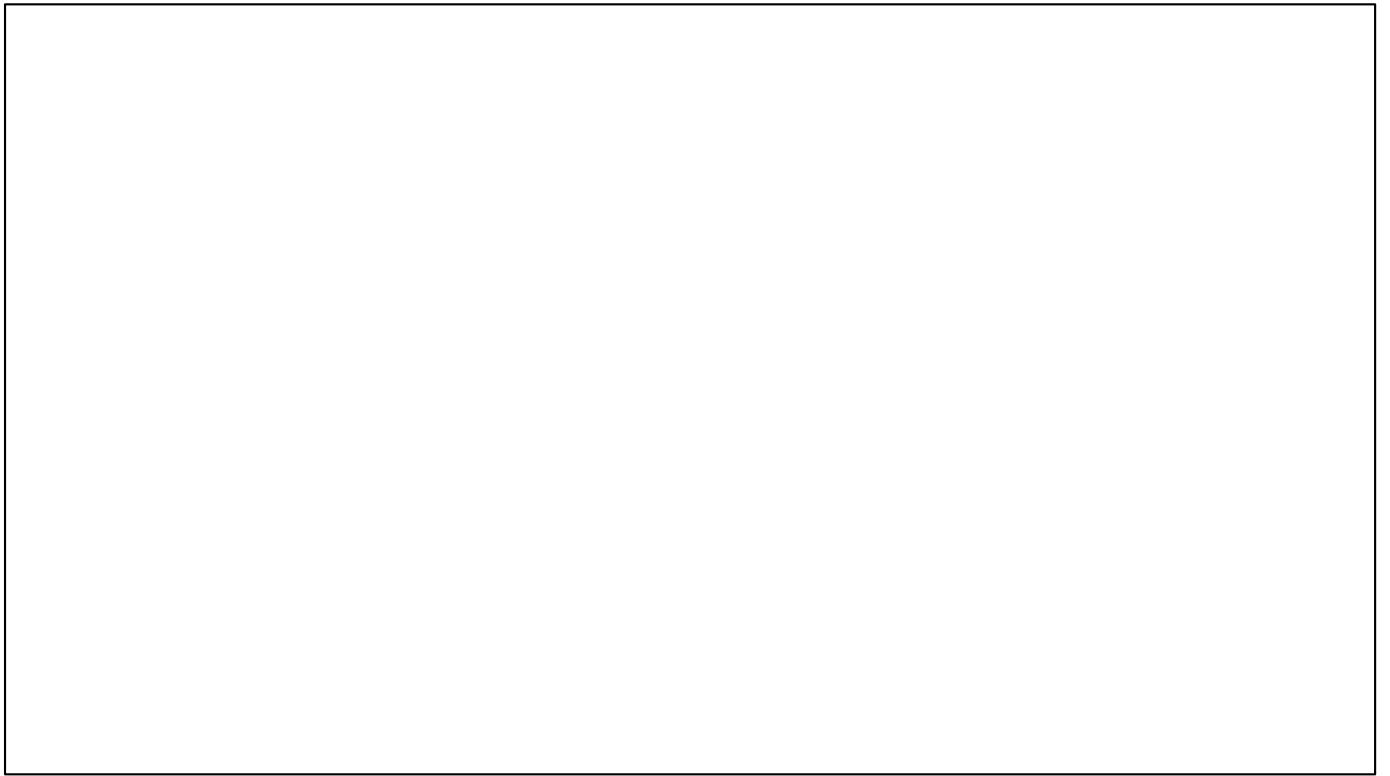
$$\begin{array}{r}
 010001 \\
 \hline
 10001
 \end{array}$$

$$\Rightarrow 17$$

$$\begin{array}{r}
 01010 \\
 - 00111 \\
 \hline
 00111
 \end{array}$$

$$\Rightarrow 10$$

$$\Rightarrow 7$$



What is the minimum number of bits (binary digits) to represent the decimal number 32?

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

$$2^5 \rightarrow 32$$

100000

Answer: D

32 is 2^5 , that is a 1 at index 5. Index 5 is the 6th bit. 5 bits can represent the 32 numbers from 0-31 inclusive, but not 32.

How many distinct numbers can be represented in 5 bits (binary digits) ?

- A. 15
- B. 16
- C. 31
- D. 32
- E. 63

Answer: D

Since 2^5 equals 32, 5 bits can represent 32 distinct numbers.

What is the result of adding 01101 and 00100?

- A. 01001
- B. 01101
- C. 10000
- D. 10001
- E. 10101

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & & 16 & 8 & 4 & 2 & 1 \\
 & & 1 & 1 & & & \\
 & 0 & 1 & 1 & 0 & 1 & \\
 + & 0 & 0 & 1 & 0 & 0 & \\
 \hline
 1 & 0 & 0 & 0 & 1 & &
 \end{array}
 \end{array}
 \begin{array}{r}
 13 \\
 4 \\
 \hline
 17 \quad \checkmark
 \end{array}$$

Answer: D

Equivalent to $13+4 = 17$

```

11
01101
00100
-----
10001
    
```

$$0.125 + 0.25 \leq 0.375$$

Will the above expression evaluate to True (as expected)?

- A. Yes
- B. Possibly, I would have to try it out
- C. No

Answer: A

Since those values are uniquely representable as floating points numbers, i.e. as $1.0 \cdot 2^{-3}$, $1.0 \cdot 2^{-2}$ and $1.5 \cdot 2^{-2}$, there is no error in the computation and so we will know the expression will evaluate to True.